Ore-class Warm-ub!!!

Let f, g, h be three functions $R \rightarrow R$ and consider their values at 0, 1 and 2. Which of the following are logically correct statements?

a. If f, g, h are linearly independent then so are the three vectors f(0) g(0)h(0)Correct Most g(1) f(1) h(1)Incorrect/some g(2) f(2) h(2)

b. If f, g, h are dependent, then so are the three vectors g(0) f(0)h(0)

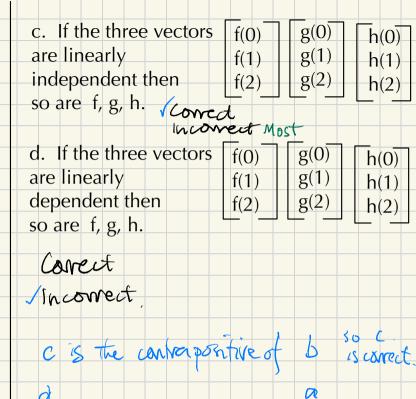
g(1)

f(1)

f(2)

g(2) h(2)Correct More IF af + bg + ch = 0 fundin In correct fewer then a f(0) + bg(0) + ch(0) = 0 (f(0)] $so a \frac{f(i)}{f(i)} + b g(i)$

h(1)



We used c. last time with ex, sinx, (

(q(D)

h(0)

t C h(1) = 0

Section 5.1: second order linear differential equations Section 5.2: higher order linear differential equations

These two sections do the same thing.

Vocabulary review:

- Linear, homogeneous differential equations
- Solution space, initial value problem
- Linearly independent solutions

New vocabulary

- superposition of solutions = the solution +
- characteristic equation

= the solution to a homogeneous a point on form a rector that

We learn:

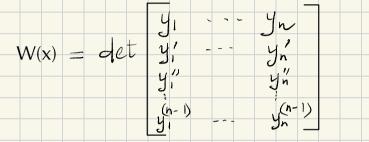
- The Wronskian
- Solutions of linear homogeneous d.e.'s form a vector space.
- How to use the characteristic equation to solve homogeneous equations with constant coefficients
- What to do about non-homogeneous equations: complementary functions.

Question 5 from Section 5.1 (like questions 3 $c_1 + c_2 =$ and 10 on the HW). $1 Z C_2$ $c_1 + 2c_2 = 0$ Given two solutions $C_1 = \frac{\begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}}{\begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}}$ $y_1 = e^x \text{ and } y_2 = e^{2x}$ $C_2 = |i \circ|$ 10mogeneous of the equation y'' - 3y' + 2y = 0linear, 2nd order find a particular solution with y(0) = 1 and y'(0) = 02 - 2 - 1 $C_{1} = \frac{2}{2} = 2$ y, = et and y2 = e^{2x} are solutions Here, all functions c, y, + c, ye The particular solution is where ci, cz are numbers are $y=2e^{x}-e^{2}$ solutions, We find c1, C2. Note $(c_1y_1 + c_2y_2)' = c_1e^{x} + 2c_2e^{2x}$ $c_1 y_1 + c_2 y_2 = c_1 e^{\chi} + c_2 e^{2\chi}$ Put x = 0

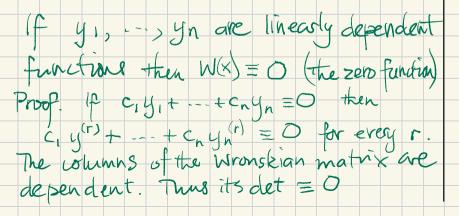
Independence using the Wronskian

Definition

Suppose we are given n functions y_1, \dots, y_m Their Wronskian is the function



Theorem (easy and useful part of bigger theorem)



Corollary. If W(x) = 0 then y12 .- 2 yn are independent,

5.1 question 26 (like question 25) Show that $2\cos x + 3\sin x$, $3\cos x - 2\sin x$ are independent.

Solution. $2\cos x + 3\sin x$ $3\cos x - 2\sin x$ $W(x) = -2\sin x + 3\cos x$ $-3\sin x - 2\cos x$ = $(2\omega x + 3sm x)(-3sm x - 2\omega x)$ - (-25(hx+3uxx) (3ux - 25 hx) = $(-9 - 4) \sin^2 x + (-4 - 9) \cos^2 x + (-12 + 12) \sin x \cos x$ $= (-13)(\sin^2 x + \cos^2 x) = -13 \neq 0$ Thus the functions are independent.

Example done for section 4.7.

Are the functions e^x , sin x and 1 linearly independent?

Solution $W(x) = e^{X} + 51h^{X} + 1$ $W(x) = e^{X} + 105 \times 0$ $e^{X} + 51h^{X} + 0$



Thus the functions are independent.

Pre-class Warm-up!!!

Which of the following statements did we prove last time?

Let y_1, \ldots, y_n be functions of x and let W(x) be their Wronskian.

 \sqrt{a} . If y_1, ..., y_n are dependent then W(x) is identically zero.

b. If W(x) is identically zero then y_1, \ldots, y_n are dependent.

c. If y_1, \ldots, y_n are independent then W(x) is not identically zero

 \sqrt{d} . If W(x) is not identically zero then y_1, ..., y_n are independent

e. We didn't prove any of these last time.

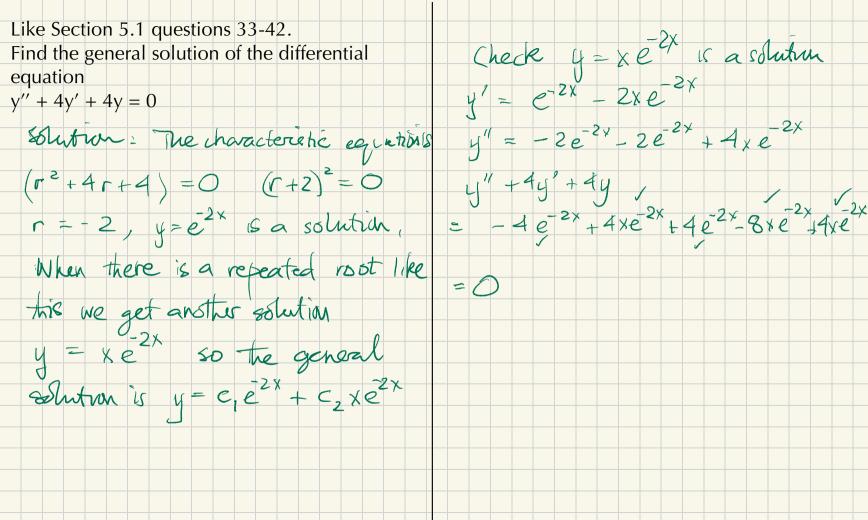
Can we even remember what the Wronstrian is 5

f. Are we comfortable making an adjective into a noun 3

Theorems about the existence of solutions Take an independent set of solutions y, y, To get a solution satisfying The following theorem combines from Section 5.1: Theorems 1, 4 the initial conditions we solve from Section 5.2: Theorems 1, 4. $\begin{array}{c|c} y_{1}(a) & y_{2}(a) & \cdots & y_{r}(a) \\ y_{1}(a) & y_{2}(a) & \cdots & y_{r}(a) \\ \vdots & \vdots & \vdots \\ y_{1}(a) & y_{2}(a) & \cdots & y_{r}(a) \\ \vdots & \vdots & \vdots \\ y_{r}(a) & \vdots & y_{r}(a) \\ \vdots & \vdots & \vdots \\ y_{r}(a) & \vdots & y_{r}(a) \\ \vdots & \vdots \\ \vdots & \vdots$ Theorem. The solutions to the nth order linear d.e. $y^{(n)} + p_{n}(x)y^{(n-1)} + \dots + p_{n-1}(x)y' + p_{n}(x)y = 0$ where p_1, ..., p_n are continuous form a vector space of dimension n. IF there is a solution, it is unique. This implies r ≤ n. Now assume that Theorem 2. For each number a and for all numbers $b_0, \ldots, b_{(n-1)}$, there is a unique y, ,..., yr is a maximal independent set. solution y with $y(a) = b_0, y'(a) = b_1, \dots, y^{(n-1)}(a) = b_{(n-1)}$ It is a basis, and so y,, ..., yr span the Assuming theorem 2, prove theorem solution space. There always is a solution that the space of solutions has dimension to the matrix equation, so r >n n -We conclude r=n.

Solving linear differential equations with constant coefficients: the characteristic equation These look like ay'' + by' + cy = 0 where a, b, c are numbers. Look for solutions of form y = ex $y' = re^{rx}$ $y'' = re^{rx}$ Substitute $ar^2e^{rx} + bre^{rx} + Ce^{rx} = 0$ $e^{rx}(qr^2+br+c)=0$. We get solution when $ar^2 + br + c = O$. This is the characteristic equation

Like Section 5.1 questions 33-42. Find the general solution of the differential equation y'' - 2y' - 3y = 0Solution. The characteristic equation is $r^{2} - 2r - 3 = 0$ (r - 3)(r + 1) = 0r=3 or r=-1y=e y=ex are solutions. The general solution is $Y = C_1 e^{3X} + C_2 e^{X}$



A stronger theorem about the Wronskian

Theorem. Suppose the n function y_1, ... y_n are solutions of a homogeneous nth order linear d.e. with continuous coefficients of

y, ... , y^(n−1).

a. If they are dependent then their Wronskian is identically 0. We and this already

b. If they are independent then their Wronskian is never 0.

For a proof of b. using 'Abel's formula' see 5.1 question 32 and 5.2 question 35.

Particular and complementary solutions

Like 5.2 questions 21-24

24. A non-homogeneous d.e., a complementary solution y_c and a particular solution y_p are given. Find a solution satisfying the initial conditions:

y'' - 2y' + 2y = 2x, y(0) = 4, y'(0) = 8

 $y_{c} = c_{1}e^{x}\cos x + c_{2}e^{x}\sin x$ $y_{f} = y_{f} + y_{f} = c_{1}e^{x}\cos x - c_{1}e^{x}\sin x$ $y_{f} = x + 1$ $y_{f} = x + 1$ $y_{f} = c_{1} + c_{2} + 1 = 8$ $y_{f} = c_{1} + c_{2} + 1 =$

A particular solution is an actual solution

The general solution has the form

$$y_c + y_p$$
 where y_c is a
general solution to the homogeneous
 cqn .
Solve $y(0) = y_c(0) + y_p(0)$
 $= c_1 + 1 = 4$, $c_1 = 3$
 $y' = y'_c + y'_p = c_1 e^{\zeta_1} os x_{-} c_1 e^{\zeta_2} sin x$
 $+ c_2 e^{\zeta_2} sin x + c_2 e^{\zeta_2} cos x$
 $y'(0) = c_1 + c_2 + 1 = 8$ so $c_2 = 4$
M The solution is

Non-homogeneous equations

Let
$$y^{(n)} + p(x)y^{(n-1)} + \dots + p_n(x)y = f(x)$$

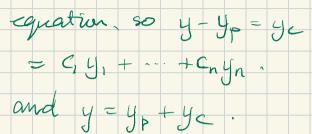
be a non-homogeneous nth order linear d.e. The associated homogeneous equation is the same, with 0 on the right instead of f(x).

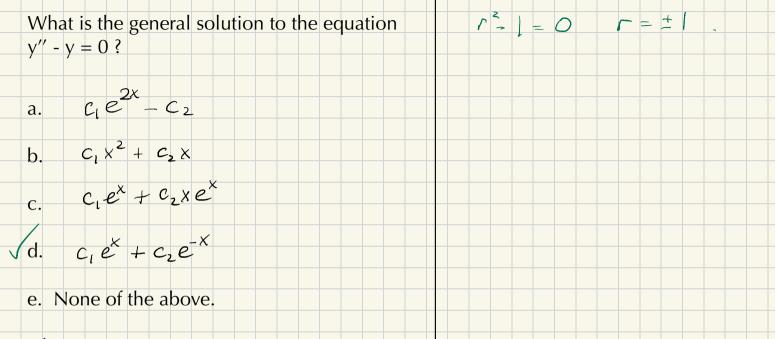
Suppose the p_i are continuous.

Theorem 5.

Let y_p be a (particular) solution of the nonhomogeneous equation, and let y_1, ... y_n be a basis for the solution space of the associated homogeneous equation. Then every solution of the non-homogeneous equation can be written $y_p + y_c$

where $y_c = c_1y_1 + ... + c_ny_n$ is a solution of the associated homogeneous equation.





where c_1, c_2 are constants.